

A study on the relations between the topological parameter and entanglement

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In this paper, some relations between the topological parameter d and concurrences of the projective entangled states have been presented. It is shown that for the case with $d = n$, all the projective entangled states of two n -dimensional quantum systems are the maximally entangled states (i.e. $C = 1$). And for another case with $d \neq n$, C both approach 0 when $d \rightarrow +\infty$ for $n = 2$ and 3. Then we study the thermal entanglement and the entanglement sudden death (ESD) for a kind of Yang-Baxter Hamiltonian. It is found that the parameter d not only influences the critical temperature T_c , but also can influence the maximum entanglement value at which the system can arrive at. And we also find that the parameter d has a great influence on the ESD.

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I. INTRODUCTION

Quantum Entanglement(QE)[1], the most surprising nonclassical property of quantum system, provides a fundamental resource in realizing quantum information and quantum computers [2] and is widely exploited in quantum cryptography[3], dense coding, teleportation [4]. It has been clarified that the entanglement of a quantum state is one of the most important properties not only in quantum information science but also in condensed matter physics. The thermal entanglement has been investigated in the system of the Heisenberg XXX [5, 6], XX [7], XXZ [8], and the Ising [9] models. Recently in Ref.[10] it has been shown that there exists a certain class of two-qubit states which display a finite entanglement decay time. This phenomenon is aptly called ESD and cannot be predicted from quantum decoherence which is an asymptotic phenomenon. It has received a lot of attentions both theoretically and experimentally[10–14].

The Temperley-Lieb algebra(TLA) first appeared in statistical mechanics as a tool to analyze various interrelated lattice models[15] and was related to link and knot invariants[16]. Either algebraically by generators and relations as in Jones original presentation[17], or as a diagram algebra modulo planar isotopy as in Kauffmans presentation[18], the TLA has always hitherto been presented as a quotient of some sort. Recently in Ref.[19], the TLA is found to present a suitable mathematical framework for describing quantum teleportation, entangle swapping, universal quantum computation and quantum computation flow. In a very recent work[20], Abramsky traced some of the surprising and beautiful connections from knot theory to logic and computation via quantum mechanics. However, the physical meaning of the important topological parameter d (describing the unknotted loop \bigcirc in topology) is still unclear. Motivated by this, in this paper we focus on studying the relations between the parameter d and entanglement to explore what role do the parameter d play in the entanglement.

The paper is organized as follows: In Sec.2, we study the relations between the topological parameter d and concurrences of the projective entangled states. It is shown that for the case with $d = n$, all the projective entangled states of two n -dimensional quantum systems are the maximally entangled states (i.e. $C = 1$). And for another case with $d \neq n$, C both approach 0 when $d \rightarrow \infty$ for $n = 2$ and 3. In Sec.3, the thermal entanglement for a kind of Yang-Baxter Hamiltonian related to the TLA is investigated. We find that the parameter d has great influences on the thermal entanglement. It not only influences the critical temperature T_c , but also can influence the maximum entanglement value at which the system can arrive at. In Sec.4, the ESD for the same Yang-Baxter Hamiltonian is investigated. It is found that the parameter d has a great influence on the ESD. A summary is given in the last section.

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FIG. 1: Diagrammatic the TLA

II. SOME RELATIONS BETWEEN THE PARAMETER d AND C

In this section, we first obtain the projective entangled state $|\Psi_{i,i+1}\rangle$ of two n -dimensional quantum systems, which contains the topological parameter d . Then we come to investigate the concurrences C of the states $|\Psi_{i,i+1}\rangle$ to explore some relations between the parameter d and C .

In order to keep the paper self-contained, we first briefly review the theory of the TLA[15]. It is a unital algebra generated by U_i ($i = 1, 2, \dots, N - 1$) which satisfy the following relations,

$$\begin{aligned} U_i^2 &= dU_i \\ U_i U_j U_i &= U_i & |i - j| = 1 \\ U_i U_j &= U_j U_i & |i - j| > 1 \end{aligned} \quad (1)$$

where d ($d \in \mathbb{C}$ and $d \neq 0$) is the unknotted loop \bigcirc in the knot theory which does not depend on the sites of the lattices. The notation $U_i \equiv U_{i,i+1}$ is used, $U_{i,i+1}$ is short for $1_1 \otimes \dots \otimes 1_{i-1} \otimes U_{i,i+1} \otimes 1_{i+2} \otimes \dots \otimes 1_N$, and 1_j represents the unit matrix of the j -th particle. These relations are diagrammatically represented in Fig. 1.

In the following, we write the $n^2 \times n^2$ ($n = 2, 3, \dots, n$) matrix U as a form of projectors in the tensor product of two nearest n -dimensional quantum spaces as follows,

$$U_{i,i+1} = d|\Psi_{i,i+1}\rangle\langle\Psi_{i,i+1}|, \quad (2)$$

where $|d|^{\frac{1}{2}}|\Psi_{i,i+1}\rangle$ describes \bigcup and $|d|^{\frac{1}{2}}\langle\Psi_{i,i+1}|$ describes \bigcap in topology. Although the parameter d can be arbitrary, in this paper we restrict ourselves on $d > 0$ for convenience. The projective entangled state $|\Psi_{i,i+1}\rangle$ of two n -dimensional quantum systems, which contains the topological parameter d , takes of the following form,

$$|\Psi_{i,i+1}\rangle = \sum_{\lambda,\mu=0}^{n-1} \alpha_{\lambda\mu} |\lambda\rangle_i |\mu\rangle_{i+1}. \quad (3)$$

where $|\lambda\rangle_i$ and $|\mu\rangle_{i+1}$ are the orthonormal bases of the Hilbert spaces i and $i + 1$ respectively, and $\alpha_{\lambda\mu}$'s are complex numbers satisfying the normalization condition $\sum_{\lambda,\mu=0}^{n-1} |\alpha_{\lambda\mu}|^2 = 1$. And we set in each row λ and each column μ of the matrix α there is a single nonzero element. The generators can be written as,

$$(U_{i,i+1})_{\lambda' \mu'}^{\lambda \mu} = d \alpha_{\lambda\mu} \alpha_{\lambda' \mu'}^* \quad \lambda, \mu, \lambda', \mu' = 0, 1, 2, \dots, n - 1. \quad (4)$$

By calculation, it is easy to see that the first relation of Eq.(1) is automatically satisfied. In order to satisfy the second relation of Eq.(1), the fulfilled conditions read,

$$\begin{cases} d^2 \sum_{\lambda,\nu,\sigma=0}^{n-1} \alpha_{\nu\lambda}^* \alpha_{\lambda\mu} \alpha_{\nu\sigma} \alpha_{\sigma\beta}^* = \delta_{\mu\beta}, \\ d^2 \sum_{\lambda,\nu,\sigma=0}^{n-1} \alpha_{\mu\lambda} \alpha_{\lambda\nu}^* \alpha_{\beta\sigma}^* \alpha_{\sigma\nu} = \delta_{\mu\beta}, \end{cases} \quad (5)$$

where $\mu = 0, 1, 2, \dots, n - 1$. By this limited conditions Eq.(5), the projective entangled state $|\Psi_{i,i+1}\rangle$ of two n -dimensional quantum systems and the corresponding topological parameter d can be determined. Via Eq.(2), the corresponding $n^2 \times n^2$ matrix U can also be obtained. Next via two classes of the parameter d , we come to study the relations between the parameter d and the concurrences C of the corresponding states $|\Psi_{i,i+1}\rangle$.

A. Example I: the case with the parameter $d = n$

In example I, we will discuss a series of the generalized $n^2 \times n^2$ ($n = 2, 3, \dots, n$) matrix U with the topological parameter $d = n$.

For the case with $\lambda = \mu$ and $\lambda' = \mu'(\lambda, \mu, \lambda', \mu' = 0, 1, 2, \dots, n-1)$ in the tensor product of two nearest n -dimensional quantum spaces, via Eq.(5) and Eq.(3), the corresponding state is

$$|\Psi\rangle = \sum_{\lambda=0}^{n-1} \frac{1}{\sqrt{n}} e^{ik_{\lambda\lambda}} |\lambda\lambda\rangle, \quad (6)$$

where the topological parameter $d = n$ and the parameters $k_{\lambda\lambda}$ are arbitrary real. By means of concurrence, we study these entangled states. In Ref.[21], the generalized concurrence (or the degree of entanglement[22]) for two qudits is given by,

$$C = \sqrt{\frac{n}{n-1}(1 - I_1)}, \quad (7)$$

where $I_1 = \text{Tr}[\rho_A^2] = \text{Tr}[\rho_B^2] = |\kappa_0|^4 + |\kappa_1|^4 + \dots + |\kappa_{n-1}|^4$, with ρ_A and ρ_B are the reduced density matrices for the subsystems, and κ_j 's ($j = 0, 1, \dots, n-1$) are the Schmidt coefficients. Then we can obtain the generalized concurrence of the state $|\Psi\rangle$ (6) as follows,

$$C = 1. \quad (8)$$

It is interesting that for the series of $n^2 \times n^2$ matrix U with the topological parameter $d = n$, all the projective states $|\Psi\rangle$ have the maximum entanglement. The state $|\Psi\rangle$ in Eq. (6) can be considered as a straightforward generalization of the symmetric Bell state $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ when $e^{ik_{\lambda\lambda}} = 1$.

B. Example II: the case with the parameter $d \neq n$

In example *II*, we will discuss another class of the $n^2 \times n^2$ matrix U with the topological parameter $d \neq n$. Because in this case we can't obtain the general generalized $n^2 \times n^2$ matrix U , we will study the cases with $n = 2$ and $n = 3$.

For the case with $n = 2$, via Eq.(5) and Eq.(3), the corresponding state is

$$|\Psi\rangle = \frac{1}{\sqrt{1+q^2}}(qe^{ik_{01}}|01\rangle + e^{ik_{10}}|10\rangle), \quad (9)$$

where the topological parameter $d = q + q^{-1}$ and $k_{01}, k_{10}, q \in \text{real}$. Hereafter, $q > 0$. The generalized concurrence of the state (9) is,

$$C = \frac{2}{d}, \quad \text{where } d \geq 2. \quad (10)$$

For the case with $n = 3$, via Eq.(5) and Eq.(3), there are three sets of solutions and the corresponding states are,

$$\begin{aligned} |\Psi\rangle^{(1)} &= \frac{1}{\sqrt{1+q+q^2}}(qe^{ik_{02}}|02\rangle + \sqrt{q}e^{ik_{11}}|11\rangle + e^{ik_{20}}|20\rangle), \\ |\Psi\rangle^{(2)} &= \frac{1}{\sqrt{1+q+q^2}}(qe^{ik_{01}}|01\rangle + e^{ik_{10}}|10\rangle + \sqrt{q}e^{ik_{22}}|22\rangle), \\ |\Psi\rangle^{(3)} &= \frac{1}{\sqrt{1+q+q^2}}(\sqrt{q}e^{ik_{00}}|00\rangle + qe^{ik_{12}}|12\rangle + e^{ik_{21}}|21\rangle), \end{aligned} \quad (11)$$

where the parameter $d = q + q^{-1} + 1$ and $k_{\lambda\mu} \in \text{real}$ ($\lambda, \mu = 0, 1, 2$). All their concurrences are the same as,

$$C = \sqrt{\frac{3}{d}}, \quad \text{where } d \geq 3. \quad (12)$$

Via these two examples, it is shown that there are some relations between the topological parameter d and concurrences C of the entangled states. In other words, the parameter d has great influences on the entanglement. Example *I* and Example *II* (i.e., $q = 1$) show that for the series of the parameter $d = n$, all the projective states $|\Psi\rangle$ of two n -dimensional quantum systems are the maximally entangled states (i.e., $C = 1$). For another class of the parameter $d \neq n$ (i.e., $q \neq 1$) in example *II*, via investigating the cases with $n = 2$ and $n = 3$, it is found that the concurrences C both decrease when d goes up, and it approaches 0 when $d \rightarrow +\infty$, as shown in Fig. 2. We guess that for the generalized $n^2 \times n^2$ matrix U with the parameter $d \neq n$, the conclusion, which is that when the parameter $d \rightarrow +\infty$, C approaches 0, is also correct. Another fact in Fig. 2 is that for the same value of loop d , the concurrence of the entangled two-qudit states (11) is always larger than the concurrence of the entangled two-qubit states (9). This means that the concurrence not only depends on the topological parameter d , but also depends on the dimension n .

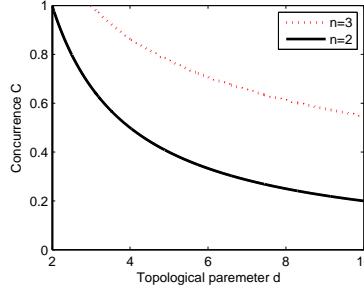


FIG. 2: The concurrence is plotted versus the parameter d . The solid line corresponds to $C = \frac{2}{d}$ for $n = 2$, and the dotted line corresponds to $C = \sqrt{\frac{3}{d}}$ for $n = 3$

III. THERMAL ENTANGLEMENT IN A YANG-BAXTER SYSTEM

In this section, we come to study the thermal entanglement for a kind of Yang-Baxter Hamiltonian, which is related to the TLA for $n = 2$, to explore the influences of the parameter d on the thermal entanglement.

By substituting Eq.(9) into Eq.(2) for $n = 2$, in the standard basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$, the 4×4 matrix U is,

$$U = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & q & e^{i\varphi} & 0 \\ 0 & e^{-i\varphi} & q^{-1} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad (13)$$

with the topological parameter $d = q + q^{-1}$ and $\varphi = k_{01} + k_{10} \in \text{real}$.

As is known, the Yang-Baxter equation (YBE)[23–25] is given by,

$$\check{R}_i(x)\check{R}_{i+1}(xy)\check{R}_i(y) = \check{R}_{i+1}(y)\check{R}_i(xy)\check{R}_{i+1}(x), \quad (14)$$

where x and y are spectrum parameters. Via the trigonometric Yang-Baxterization approach[26], it gives

$$\begin{aligned} \check{R}(x) &= [q^2 + q^{-2} - (x^2 + x^{-2})]^{-\frac{1}{2}}[(qx - q^{-1}x^{-1})I - (x - x^{-1})U], \\ \check{R}^{-1}(x) &= [q^2 + q^{-2} - (x^2 + x^{-2})]^{-\frac{1}{2}}[(qx^{-1} - q^{-1}x)I + (x - x^{-1})U]. \end{aligned} \quad (15)$$

It is easy to check that $\check{R}^+(x) = \check{R}^{-1}(x) = \check{R}(-x)$ for $x = e^{i\theta}$, where $\theta \in \text{real}$. It is worth to mention that in this paper, the real parameters θ and φ are time-independent.

Here we study a original Hamiltonian describing two spin-1/2 particles (particle 1 and 2) interaction,

$$H_0 = \mu_1 S_1^z + \mu_2 S_2^z + g S_1^z S_2^z,$$

where μ_i ($i = 1, 2$) represent external magnetic field and g is the interaction of z -component of two-qubit spins. Taking the Schrödinger equation $i\hbar\partial|\Psi\rangle/\partial t = H|\Psi\rangle$ into account, where $|\Psi\rangle = \check{R}(x)|\Psi_0\rangle$ and $|\Psi_0\rangle$ is the eigenstate of H_0 , one can get a new Hamiltonian as $H(\theta, \varphi) = \check{R}(x)H_0\check{R}^{-1}(x)$ [27], where the real parameters θ and φ are time-independent. For convenience, we let $x = i$ (i.e., $\theta = \frac{\pi}{2}$). Then we arrive at a new Hamiltonian,

$$H = (B + J(1 - \frac{8}{d^2}))S_1^z + (B - J(1 - \frac{8}{d^2}))S_2^z + gS_1^z S_2^z - \frac{4J\sqrt{d^2 - 4}}{d^2}(e^{i\varphi} S_1^+ S_2^- + e^{-i\varphi} S_1^- S_2^+), \quad (16)$$

where $B = \frac{\mu_1 + \mu_2}{2}$ and $J = \frac{\mu_1 - \mu_2}{2}$, and $S_i^\pm = S_i^x \pm iS_i^y$ are raising and lowering operators respectively for the i -th particle. Specifically, we find that when $\varphi = \pi$, this model is the two-qubit anisotropic Heisenberg XXZ model under an inhomogeneous magnetic field. $B \geq 0$ is restricted, and the magnetic fields on the two spins have been so parameterized that $J(1 - 8/d^2)$ controls the degree of inhomogeneity. For the system (16), its corresponding eigenstates read $|\Psi_1\rangle = |00\rangle$, $|\Psi_2\rangle = |11\rangle$, $|\Psi_3\rangle = \frac{2}{d}(\frac{-\sqrt{d^2-4}e^{i\varphi}}{2}|01\rangle + |10\rangle)$, $|\Psi_4\rangle = \frac{2}{d}(|01\rangle + \frac{\sqrt{d^2-4}e^{-i\varphi}}{2}|10\rangle)$, with corresponding energies $E_1 = B + \frac{q}{4}$, $E_2 = -B + \frac{q}{4}$, $E_3 = J - \frac{q}{4}$, $E_4 = -J - \frac{q}{4}$.

Next to quantify the entanglement of formation of a mixed state ρ of two qubits, we use the Wootters concurrence [28] defined as,

$$C(t) = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}, \quad (17)$$

where $\{\lambda_i\}$ are the eigenvalues of the matrix $\rho(\sigma_y^A \otimes \sigma_y^B)\rho^*(\sigma_y^A \otimes \sigma_y^B)$, with ρ^* denoting complex conjugation of the matrix ρ and $\sigma_y^{A/B}$ are the Pauli matrices for atoms A and B. When spin chains are subjected to environmental disturbance, they inevitably become thermal equilibrium states. The thermal state at finite temperature T is $\rho(T) = \frac{1}{Z} \exp(-\frac{H}{kT})$, where $Z = \text{Tr}[\exp(-\frac{H}{kT})]$ is the partition function and k is the Boltzmann constant. For simplicity, we write $k = 1$. By calculation, the density matrix $\rho(T)$ of the system (16) can be written as,

$$\rho(T) = \frac{1}{2(\cosh \frac{B}{T} + e^{\frac{g}{2T}} \cosh \frac{J}{T})} \begin{pmatrix} e^{\frac{-B}{T}} & 0 & 0 & 0 \\ 0 & e^{\frac{g}{2T}} (\cosh \frac{J}{T} - (1 - \frac{8}{d^2}) \sinh \frac{J}{T}) & \frac{4\sqrt{d^2-4}}{d^2} e^{\frac{g}{2T}} \sinh \frac{J}{T} e^{i\varphi} & 0 \\ 0 & \frac{4\sqrt{d^2-4}}{d^2} e^{\frac{g}{2T}} \sinh \frac{J}{T} e^{-i\varphi} & e^{\frac{g}{2T}} (\cosh \frac{J}{T} + (1 - \frac{8}{d^2}) \sinh \frac{J}{T}) & 0 \\ 0 & 0 & 0 & e^{\frac{B}{T}} \end{pmatrix}. \quad (18)$$

The concurrence is calculated as,

$$C = \max \left(\frac{\frac{4\sqrt{d^2-4}}{d^2} e^{\frac{g}{2T}} \sinh \frac{|J|}{T} - 1}{\cosh \frac{B}{T} + e^{\frac{g}{2T}} \cosh \frac{J}{T}}, 0 \right). \quad (19)$$

Now we do the limit $T \rightarrow 0$ on the concurrence (19), we obtain,

$$\begin{aligned} \lim_{T \rightarrow 0} C_1 &= \frac{4\sqrt{d^2-4}}{d^2} \quad \text{for} \quad |B| > |J| + \frac{g}{2}, \\ &= \frac{2\sqrt{d^2-4}}{d^2} \quad \text{for} \quad |B| = |J| + \frac{g}{2}, \\ &= 0 \quad \text{for} \quad |B| < |J| + \frac{g}{2}. \end{aligned} \quad (20)$$

It is worth to mention that the influences of the parameters g and B on the thermal entanglement have been discussed in our paper[27], whose model corresponds to the topological parameter $d = 2$ (i.e., $q = 1$). Here we emphasize on exploring the parameter d 's influences on the thermal entanglement. From Eq. (20) we can see that at $T = 0$, the entanglement vanishes as $|B|$ crosses the critical value $|J| + \frac{g}{2}$, which means that the critical magnetic field B_c is independent on the parameter d . An important point revealed by Eq. (20) is that the maximum entanglement value at which the system can arrive at, which is $C_{max} = \frac{4\sqrt{d^2-4}}{d^2}$ for $|B| > |J| + \frac{g}{2}$ at $T = 0$, is dependent on the parameter d . Fig. 3 shows that when the parameter $d = 2\sqrt{2}$, the ground states $|\Psi_3\rangle = \frac{1}{\sqrt{2}}(-|01\rangle + |10\rangle)$ or $|\Psi_4\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$ both become the maximally entangled states, so the maximum entanglement value $C_{max} = 1$. When the parameter $d \rightarrow 2$ or $d \rightarrow +\infty$, the ground states $|01\rangle$ or $|10\rangle$ both have no entanglement, then the maximum entanglement value $C_{max} = 0$. Another important character revealed by Eq. (19) is that the critical temperature T_c , which is determined by the nonlinear equation $\frac{4\sqrt{d^2-4}}{d^2} e^{\frac{g}{2T_c}} \sinh \frac{|J|}{T_c} = 1$, is also dependent on the parameter d . From Fig. 4, it is shown that when $d = 2\sqrt{2}$, T_c arrive at the maximum value (i.e., T_c is about 1.5). When $d \rightarrow 2$ or $d \rightarrow +\infty$, all the four eigenstates are unentangled states, so the critical temperature $T_c = 0$. Thus we can obtain a higher entanglement at a fixed temperature via changing the values of the parameter d .

IV. ESD IN THE SAME YANG-BAXTER SYSTEM

In this section, we study the ESD in the same Yang-Baxter system (16) to explore the influence of the parameter d on the ESD.

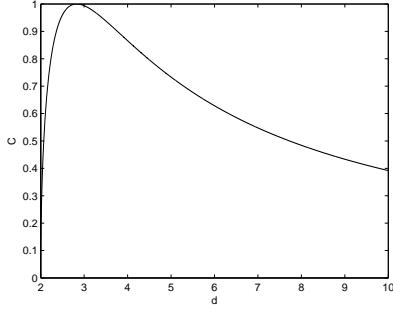


FIG. 3: The maximum entanglement value C_{max} is plotted versus the parameter d .

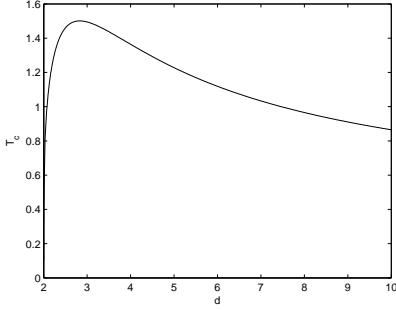


FIG. 4: The critical temperature T_c is plotted versus the parameter d . Coupling constant $J = 1$ and the parameter $g = 1$.

The time evolution $U(t) = \exp\{-iHt\}$ is written in the basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$,

$$\begin{aligned}
 U_{11} &= e^{-i(B+\frac{g}{4})t} \\
 U_{44} &= e^{i(B-\frac{g}{4})t} \\
 U_{22} &= e^{i\frac{g}{4}t}(\cos[Jt] - i(1 - \frac{8}{d^2})\sin[Jt]), \\
 U_{33} &= e^{i\frac{g}{4}t}(\cos[Jt] + i(1 - \frac{8}{d^2})\sin[Jt]) \\
 U_{23} &= e^{i\frac{g}{4}t} \frac{4ie^{i\varphi}\sqrt{d^2-4}\sin[Jt]}{d^2}, \quad U_{32} = e^{i\frac{g}{4}t} \frac{4ie^{-i\varphi}\sqrt{d^2-4}\sin[Jt]}{d^2}.
 \end{aligned} \tag{21}$$

It is convenient to choose the initial state $\rho_0 = \frac{1-\gamma}{4} + \gamma|\psi\rangle\langle\psi|$ ($0 < \gamma \leq 1$) with $|\psi\rangle = \sin\alpha|01\rangle + \cos\alpha|10\rangle$. It is worth to mention that in our paper[29], it has been shown that in Yang-Baxter systems, the ESD is not only sensitive to the initial condition, but also has relations with the different Yang-Baxter systems. And it has been found that the meaningful parameter φ has a great influence on the ESD. Here we emphasize on studying the influences of the unknotted loop d on the ESD. For convenience, we let the parameters $\alpha = \frac{\pi}{4}$, $\gamma = 0.5$, $J = \frac{1}{2}$ and $\varphi = \pi$. The system model (16) corresponds to the two-qubit anisotropic Heisenberg XXZ model under an inhomogeneous magnetic field. Then the entanglement for $\rho(t) = U(t)\rho_0U^+(t)$ can be given easily, and according to Eq.(17), the concurrence can be obtained as follows,

$$C = \frac{\sqrt{(16(d^2-4) + (d^2-8)^2\cos t)^2 + d^4(d^2-8)^2\sin^2 t}}{2d^4} - \frac{1}{4}. \tag{22}$$

In Fig. 5, we give a plot of the concurrence as a function of the time t and the parameter d . It is clear that in our closed Yang-Baxter system, the ESD happens in some special times and then the entanglement revives after a while. One can note that the topological parameter d has a great influence on the ESD when the initial condition is determinate. It is obvious that the ESD happens only when the parameter d changes in a certain range. This means that in the Yang-Baxter system, one can realize the ESD via changing the values of the parameter d when the initial condition is determinate.

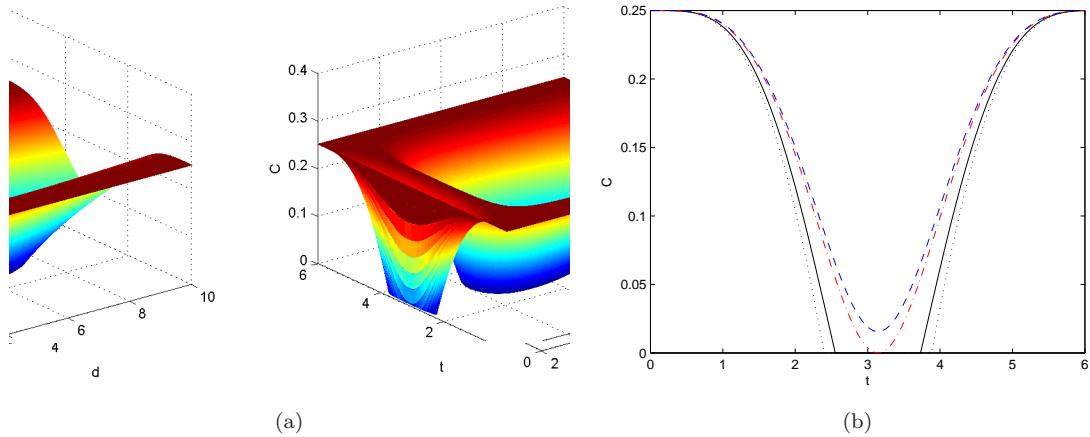


FIG. 5: The concurrence C is plotted versus the time t and the parameter d . The figure (b), the concurrences versus time t for different parameters d : $d = 2.1$ (solid line), $d = 4$ (dot-dashed line), $d = 5$ (dotted line), $d = 8$ (dashed line).

V. SUMMARY

In this paper, we have presented some relations between the topological parameter d and concurrences of the projective entangled states. Specifically, it is shown that for the case with the parameter $d = n$, all the projective entangled states of two n -dimensional quantum systems are the maximally entangled states (i.e. $C = 1$). And for another case with the parameter $d \neq n$, via investigating the cases with $n = 2$ and $n = 3$, we find C both approach 0 when $d \rightarrow \infty$. Then we construct a kind of Yang-Baxter Hamiltonian related to the 4×4 matrix U , with the topological parameter $d = q + q^{-1}$ for $n = 2$. The thermal entanglement and the ESD for the Yang-Baxter system have been investigated. It is found that the parameter d has great influences on the thermal entanglement. It not only influences the critical temperature T_c , but also can influence the maximum entanglement value at which the system can arrive at. Finally we find that the parameter d also has a great influence on the ESD, and one can realize the ESD via changing the values of the parameter d when the initial condition is determinate. It is worth to mention that via our paper, it is obvious that the topological parameter d plays an important role in the entanglement.

VI. ACKNOWLEDGMENTS

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